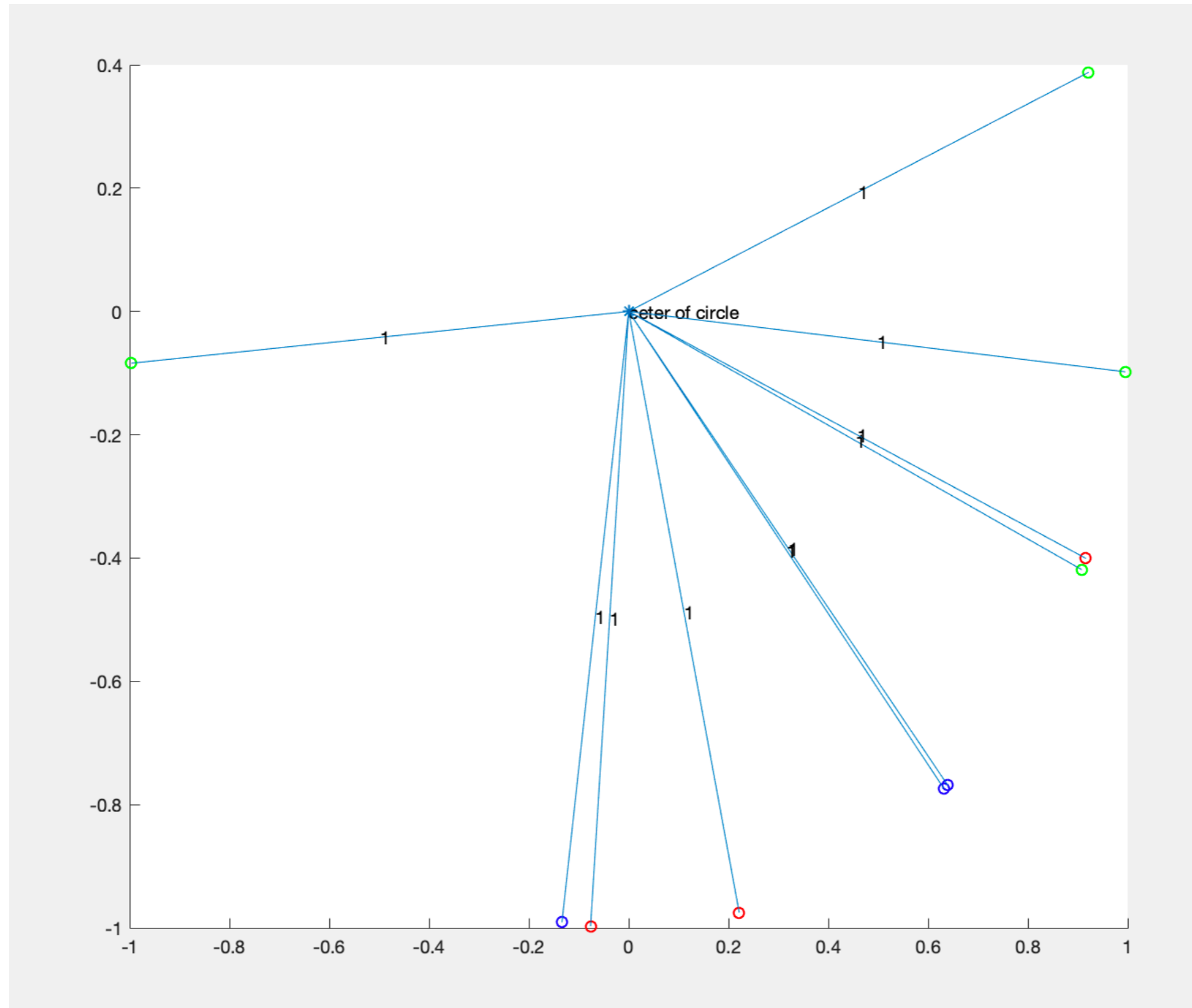
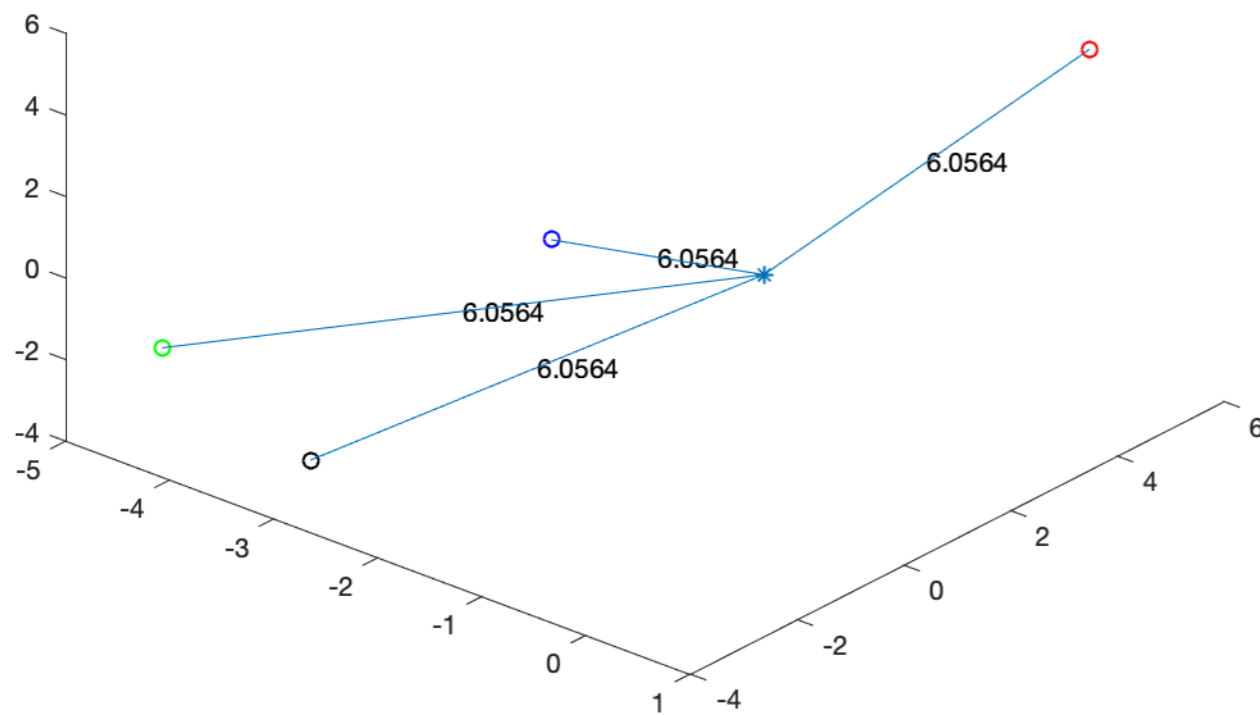


以**fsolve**解非線性方程式，並應用在三度空間不共面四點的球心計算

Find the center of m noise-added points on a circle



球心到四個點的點距離一樣



x =

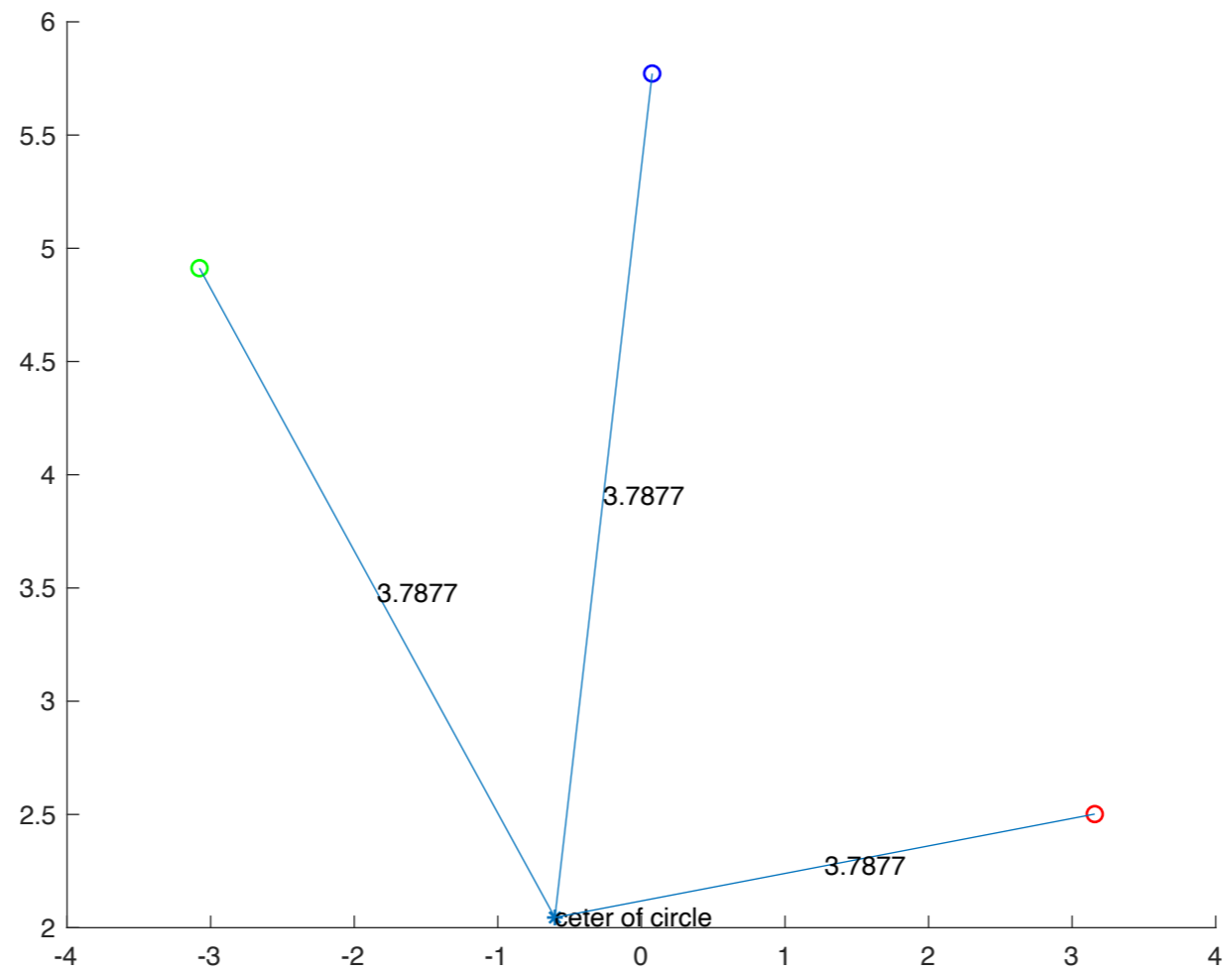
-0.9835
1.2522
0.3926

point1 r: 0.593328 4.281579 5.394286
point2 g: -4.541179 -3.087781 -1.885160
point3 b: -4.407102 3.949420 -3.812697
point4 k: -3.047241 -3.222960 -3.127972
p1 : 6.056434
p2 : 6.056434
p3 : 6.056434
p4 : 6.056434

EX0410_1

應用 `fsolve` 求根，解

二度空間不共線三點求圓心問題



X =

-0.6026
2.0450

point1 r: 3.157515 2.501672
point2 g: -3.077566 4.912235
point3 b: 0.074859 5.771626

Step 1. 在 $[-2\pi, 2\pi]^2$ 中，
隨機產生三個點

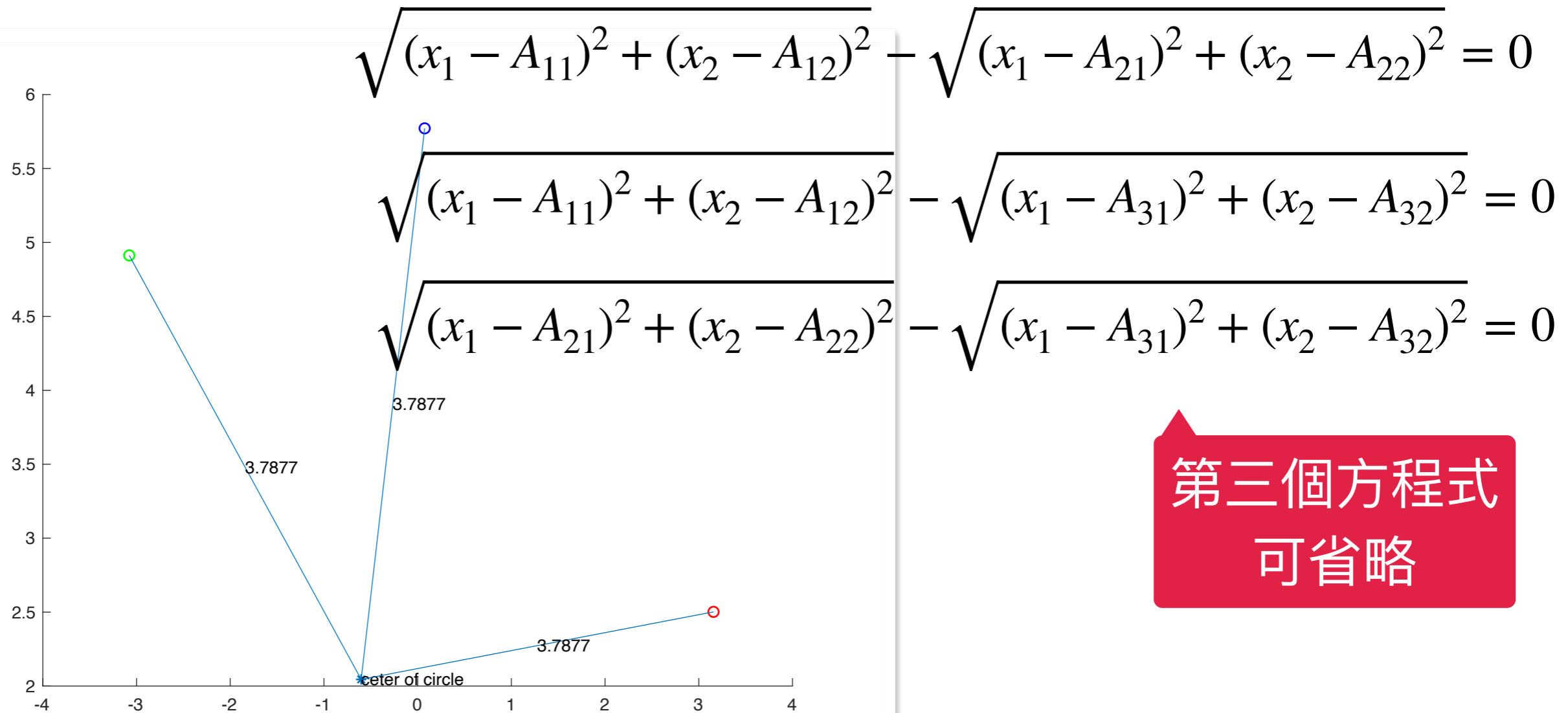
```
A = rand(3,2)*4*pi - 2*pi;
```

Step 2. 寫一個Matlab函數，計算圓心到任意兩點的距離差

```
function ans = dis(x,A)
ans(1) = sqrt((x(1)-A(1,1))^2 + (x(2)-A(1,2))^2) - sqrt( (x(1)-A(2,1))^2 + (x(2)-A(2,2))^2);
ans(2) = sqrt((x(1)-A(1,1))^2 + (x(2)-A(1,2))^2) - sqrt((x(1)-A(3,1))^2 + (x(2)-A(3,2))^2);
```

Step 3. 使用fsolve求圓心

圓心到任意兩點的距離差為0



第三個方程式
可省略

使用fsolve求函數
dis的根

```
function demo_center_circle()  
A = rand(3,2)*4*pi - 2*pi;  
x = fsolve(@(x) dis(x,A), [0;0])
```

```
function ans = dis(x,A)  
ans(1) = sqrt((x(1)-A(1,1))^2 + (x(2)-A(1,2))^2) - sqrt((x(1)-A(2,1))^2 + (x(2)-A(2,2))^2);  
ans(2) = sqrt((x(1)-A(1,1))^2 + (x(2)-A(1,2))^2) - sqrt((x(1)-A(3,1))^2 + (x(2)-A(3,2))^2);
```

$$\sqrt{(x_1 - A_{11})^2 + (x_2 - A_{12})^2} - \sqrt{(x_1 - A_{21})^2 + (x_2 - A_{22})^2} = 0$$

$$\sqrt{(x_1 - A_{11})^2 + (x_2 - A_{12})^2} - \sqrt{(x_1 - A_{31})^2 + (x_2 - A_{32})^2} = 0$$

設定[0;0]為根的初始向量

dis(x,A) 指定dis為非線性
函數，x為函數變數，A為
函數參數

```
function demo_center_circle()  
A = rand(3,2)*4*pi - 2*pi;  
x = fsolve(@(x) dis(x,A), [0;0])
```

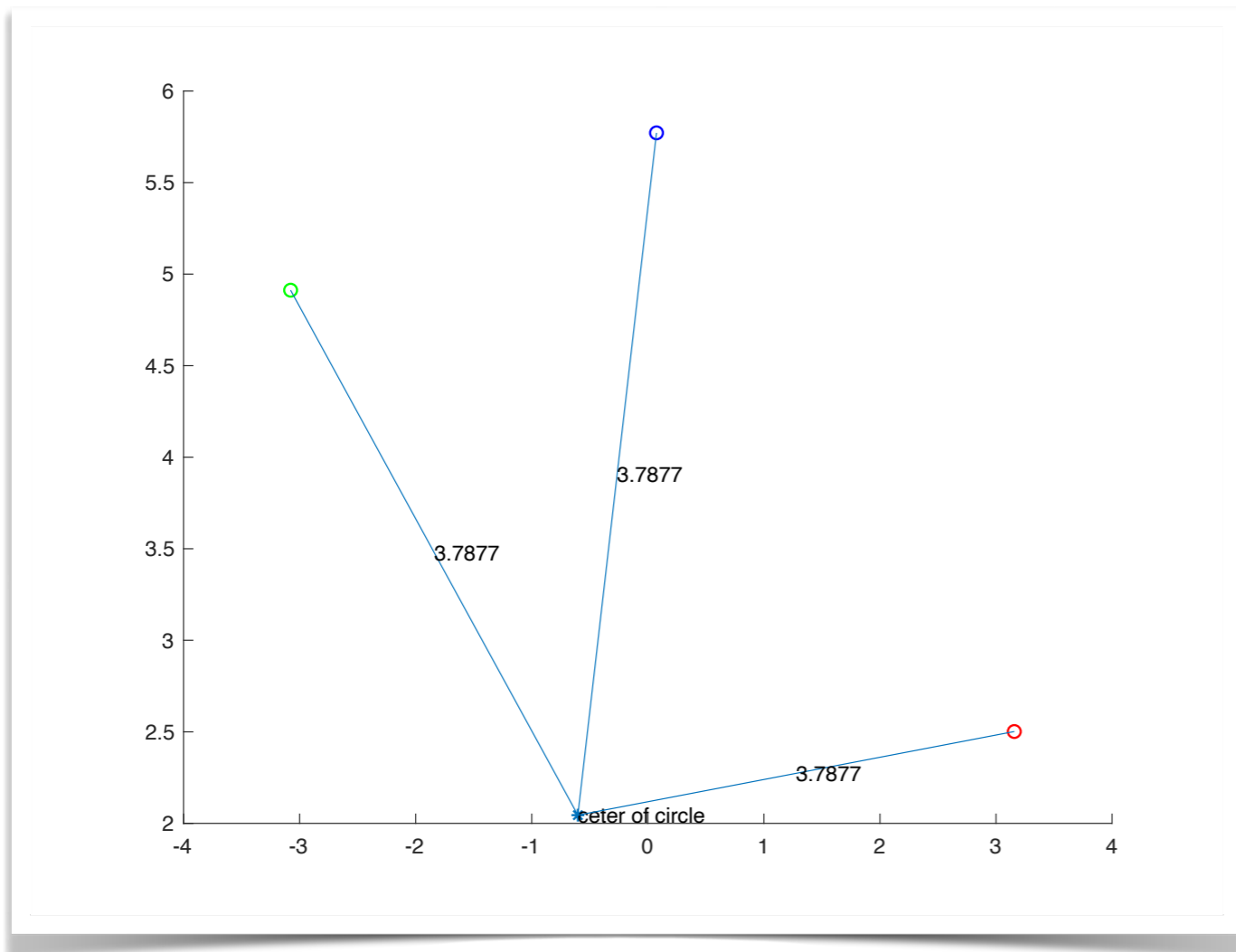
@(x) 指定x為變數

```
function ans = dis(x,A)  
ans(1) = sqrt((x(1)-A(1,1))^2 + (x(2)-A(1,2))^2) - sqrt((x(1)-A(2,1))^2 + (x(2)-A(2,2))^2);  
ans(2) = sqrt((x(1)-A(1,1))^2 + (x(2)-A(1,2))^2) - sqrt((x(1)-A(3,1))^2 + (x(2)-A(3,2))^2);
```

$$\sqrt{(x_1 - A_{11})^2 + (x_2 - A_{12})^2} - \sqrt{(x_1 - A_{21})^2 + (x_2 - A_{22})^2} = 0$$

$$\sqrt{(x_1 - A_{11})^2 + (x_2 - A_{12})^2} - \sqrt{(x_1 - A_{31})^2 + (x_2 - A_{32})^2} = 0$$

Step 4輸出並繪製點與距離



x =

-0.6026
2.0450

point1 r: 3.157515 2.501672
point2 g: -3.077566 4.912235
point3 b: 0.074859 5.771626

draw_all函數名稱，x,
A代表函數的輸入參數

分別以紅色，綠色，藍
色繪製三個點

```
function draw_all(x,A)
figure; hold on
plot(x(1),x(2),'*')
c = ['r', 'g', 'b'];
text(x(1),x(2),"center of circle")
for i = 1:3
    plot(A(i,1),A(i,2),[c(i) 'o'])
    fprintf("point%d %c: %f %f\n",i,c(i), A(i,1),A(i,2))
end
for i = 1:3
    line([x(1) A(i,1)], [x(2) A(i,2)])
    d = sqrt((x(1)-A(i,1))^2 + (x(2)-A(i,2))^2)
    text((x(1)+A(i,1))/2, (x(2)+A(i,2))/2, string(d))
end
```

列印點座標

%d代表i為整數，%c代表變數c(i)為字元

%f %f代表A(i,1)與A(i,2) 為實數

```
function draw_all(x,A)
figure; hold on
plot(x(1),x(2),'*')
c=['r','g','b'];
text(x(1),x(2),"center of circle")
for i = 1:3
    plot(A(i,1),A(i,2),[c(i) 'o'])
    fprintf("point%d %c: %f %f\n",i,c(i), A(i,1),A(i,2))
end
for i = 1:3
    line([x(1) A(i,1)], [x(2) A(i,2)])
    d = sqrt((x(1)-A(i,1))^2 + (x(2)-A(i,2))^2)
    text((x(1)+A(i,1))/2, (x(2)+A(i,2))/2, string(d))
end
```

列印線與文字

**EX0410_2應用fsolve求根，解
三度空間不共面四點求球心問題**

Step 1. 在 $[-2\pi, 2\pi]^3$ 中，
隨機產生四個點

Step 2. 寫一個Matlab函數，計算球心到任意兩點的距離差

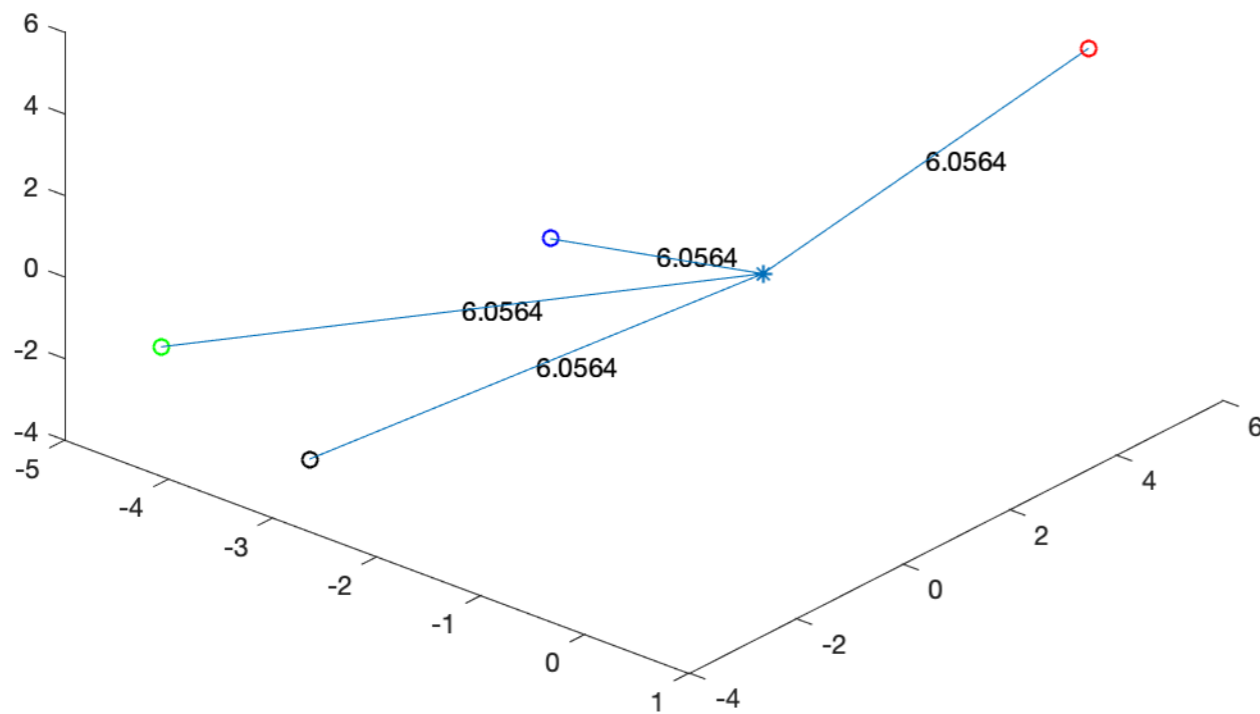
$$\sqrt{((x_1 - A_{11})^2 + (x_2 - A_{12})^2 + (x_3 - A_{13})^2)} - \sqrt{((x_1 - A_{21})^2 + (x_2 - A_{22})^2 + (x_3 - A_{23})^2)} = 0$$

$$\sqrt{((x_1 - A_{21})^2 + (x_2 - A_{22})^2 + (x_3 - A_{23})^2)} - \sqrt{((x_1 - A_{31})^2 + (x_2 - A_{32})^2 + (x_3 - A_{33})^2)} = 0$$

$$\sqrt{((x_1 - A_{31})^2 + (x_2 - A_{32})^2 + (x_3 - A_{33})^2)} - \sqrt{((x_1 - A_{41})^2 + (x_2 - A_{42})^2 + (x_3 - A_{43})^2)} = 0$$

Step 3. 使用fsolve求球心

Step 4輸出並繪製點與距離



x =

```
-0.9835  
1.2522  
0.3926
```

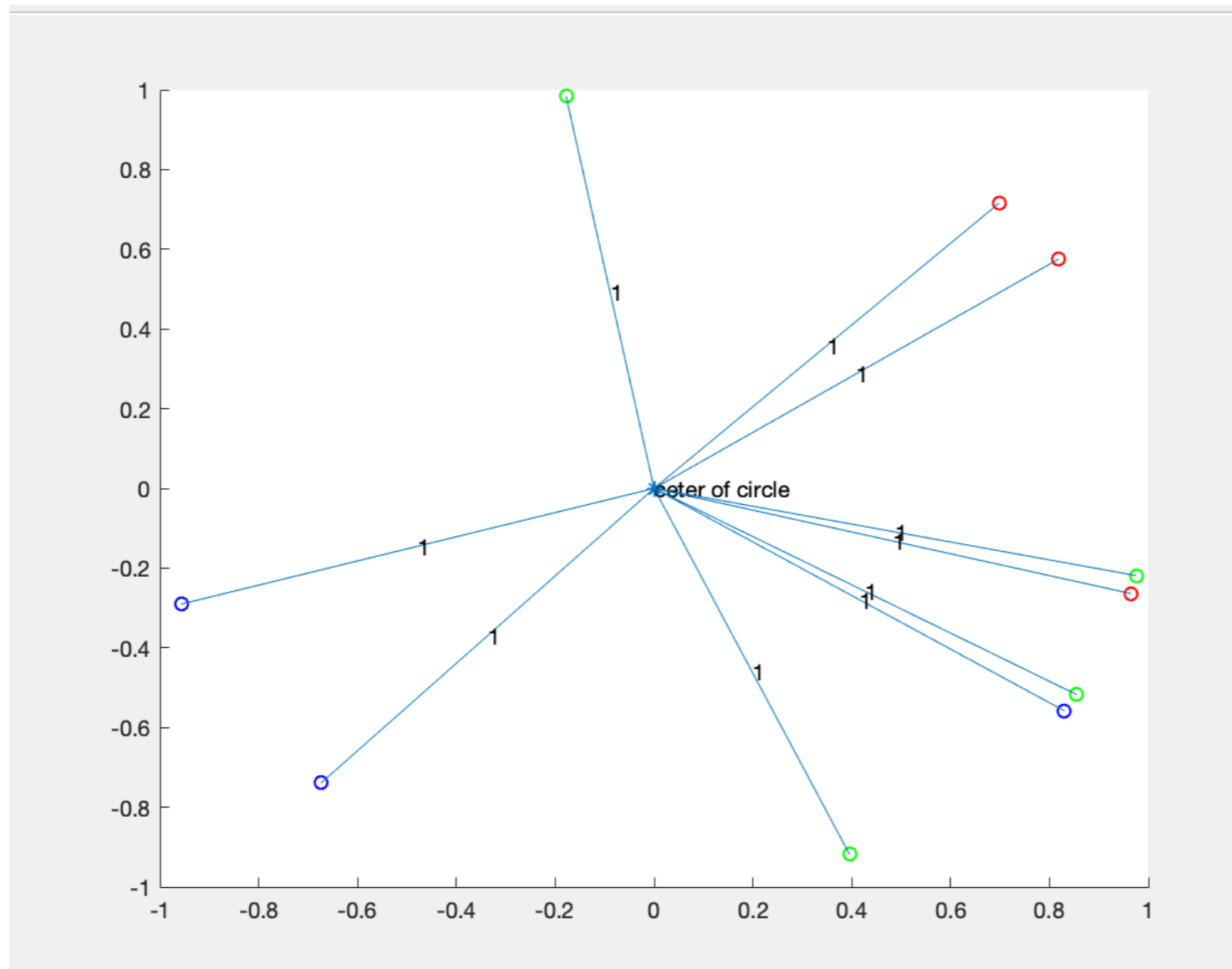
```
point1 r: 0.593328 4.281579 5.394286  
point2 g: -4.541179 -3.087781 -1.885160  
point3 b: -4.407102 3.949420 -3.812697  
point4 k: -3.047241 -3.222960 -3.127972  
p1 : 6.056434  
p2 : 6.056434  
p3 : 6.056434  
p4 : 6.056434
```

```

function draw_all(x,A)
    figure; hold on
    plot3(x(1),x(2),x(3),'*')
    c =['r','g','b','k'];
    %text(x(1),x(2),x(3),"center of outer circle")
    for i = 1:4
        plot3(A(i,1),A(i,2),A(i,3),[c(i) 'o'])
        fprintf("point%d %c: %f %f %f\n",i,c(i), A(i,1),A(i,2),A(i,3))
    end
    for i = 1:4
        line([x(1) A(i,1)],[x(2) A(i,2)],[x(3) A(i,3)])
        d = sqrt((x(1)-A(i,1))^2 + (x(2)-A(i,2))^2 + (x(3)-A(i,3))^2);
        fprintf("p%d : %f\n",i, d)
        text((x(1)+A(i,1))/2,(x(2)+A(i,2))/2,(x(3)+A(i,3))/2,string(d))
    end
end

```

Find the center of m noise-added points on a circle



```
function demo_outter_circle_m()
```

```
z = 2*pi*rand(10,1);           %create 100 random points
```

```
p = cos(z);                   %create a circle with those points
```

```
q = sin(z);
```

```
A = [p q];
```

```
x = fsolve(@(x) dis(x,A),zeros(10,1))
```

```
draw_all(x,A)
```

```
function ans = dis(x,A)
```

```
    m = size(A,1);
```

```
    for i = 1 : m-1
```

```
        ans(i) = (x(1)-A(i,1))^2 + (x(2)-A(i,2))^2 - (x(1)-A(i+1,1))^2 - (x(2)-A(i+1,2))^2;
```

```
    end
```

```
function draw_all(x,A)
    figure; hold on
    plot(x(1),x(2),'*')
    c=['r','g','b'];
    text(x(1),x(2),"center of circle")
    m = size(A,1);
    for i = 1:m
        plot(A(i,1),A(i,2),[c(mod(i,3)+1) 'o'])
        fprintf("point%d %c: %f %f\n",i,c(mod(i,3)+1), A(i,1),A(i,2))
    end
    for i = 1:m
        line([x(1) A(i,1)],[x(2) A(i,2)])
        d = sqrt((x(1)-A(i,1))^2 + (x(2)-A(i,2))^2)
        text((x(1)+A(i,1))/2,(x(2)+A(i,2))/2,string(d))
    end
end
```

EX04010_3應用fsolve 求根，解非線性系統

$$x_1^2 + x_2^2 - 1 = 0$$

$$x_1^2 - x_2^2 = 0$$

Step1. 寫一個Matlab函數，計算非線性函數輸出

$$F_1(x_1, x_2) = x_1^2 + x_2^2 - 1$$

$$F_2(x_1, x_2) = x_1^2 - x_2^2$$

Step 2. 使用fsolve求非線 性函數的根

Step 3. 將步驟2的根代入非線性函數，檢驗函數值是否接近0

EX0410_4 應用fsolve求根，解非線性系統

$$3x_1 - \cos(x_2 x_3) - \frac{1}{2} = 0$$

$$x_1^2 - 81(x_2 + 0.1)^2 + \sin(x_3) + 1.06 = 0$$

$$e^{-x_1 x_2} + 20x_3 + \frac{1}{3}(10\pi - 3) = 0$$

Step1. 寫一個Matlab函數，計算非線性函數輸出

$$3x_1 - \cos(x_2x_3) - \frac{1}{2} = 0$$

$$x_1^2 - 81(x_2 + 0.1)^2 + \sin(x_3) + 1.06 = 0$$

$$e^{-x_1x_2} + 20x_3 + \frac{1}{3}(10\pi - 3) = 0$$

Step 2. 使用fsolve求非線 性函數的根

Step 3. 將步驟2的根代入非線性函數，檢驗函數值是否接近0向量

EX0410_5應用fsolve求根，解非線性系統

$$e^{-e^{-(x_1+x_2)}} - x_2(1 + x_1^2) = 0$$

$$x_1 \cos(x_2) + x_2 \sin(x_1) - 1/2 = 0$$

Step1. 寫一個Matlab函數，計算非線性函數輸出

$$e^{-e^{-(x_1+x_2)}} - x_2(1 + x_1^2) = 0$$

$$x_1 \cos(x_2) + x_2 \sin(x_1) - 1/2 = 0$$

Step 2. 使用fsolve求非線 性函數的根

Step 3. 將步驟2的根代入非線性函數，檢驗函數值是否接近0向量