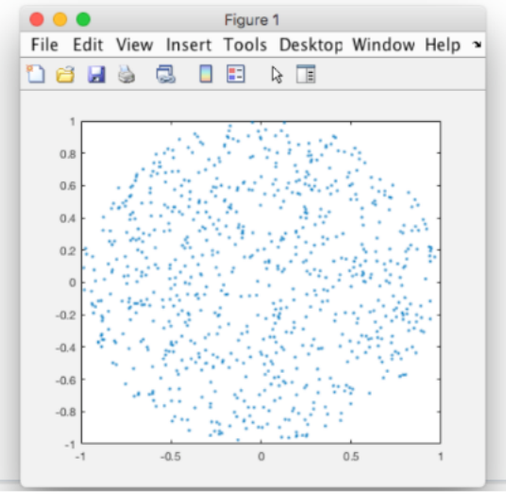
Problem set 2026

x包含1000個分佈在[-1　1]x[-1　1]的二維點座標，請計算r向量，使向量元素代表二維點與原點的距離平方，使用find函數找出向量r中小於1的元素位置，本題繪製單位圓內的二維點

```
1 -   x = rand(2,1000)*2 - 1;
2 -   r = [redacted];
3 -   ind = [redacted];
4 -   plot(x(1, ind), x(2, ind), '.')
5
6
```



(5 points) Complete the following script to evaluate factorial.

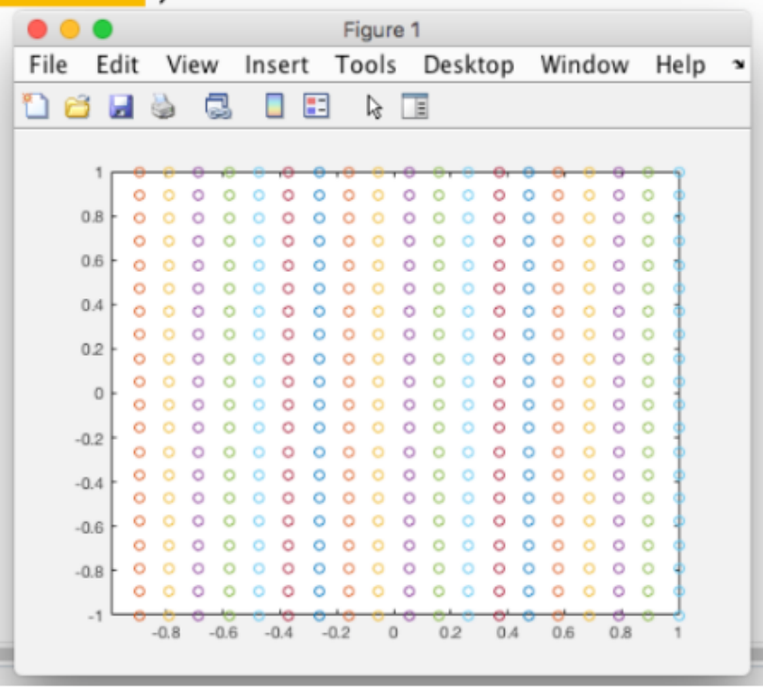
```
n = 1;
nFactorial = 1;
while nFactorial < 100
        .
        .
        .

end
fprintf('n=%d n!=%f',n,nFactorial)
```

在[–1, 1]x[–1, 1]的區域中繪製20x20的點矩陣

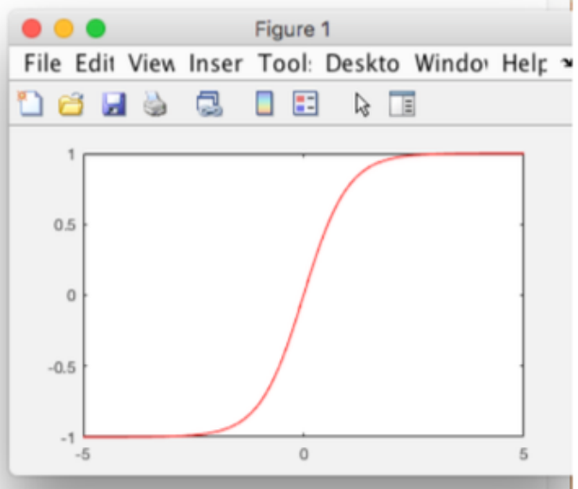
```
n = 20;
a = [redacted];
X = [redacted];
Y = [redacted];
plot(X,Y,'o')
>>
>>
>>
>>
>>
>>
>>
>>
>>
```

本題答題



本題使用switch指令，執行三種不同函數，分別是sign函數，tanh函數，以及Relu函數

```
x = linspace(-5, 5);
n = 3;
select = ceil(rand * n);
switch select
    case 1
        y = sign(x);
    case 2
        y = tanh(x);
    otherwise
        y = x;
        ind = find(y < 0);
        y(ind) = 0;
end
plot(x, y,'r')
```



matrix multiplication

```
n = 50;
A = rand(n, n);
B = rand(n, n);
C = zeros(n, n);
for i = 1 : n
    for [redacted]
        C(i, j) = [redacted];
    end
end
```

Taylor expansion

本題求tanh(a+da)的泰勒展開，展開的項數為三，請將第一項的內容儲存在變數app，並在迴圈中將da的k次方項加入變數app中

$$f(a+\delta) \sim f(a) + f'(a)\delta + \frac{1}{2!}f''(a)\delta^2$$

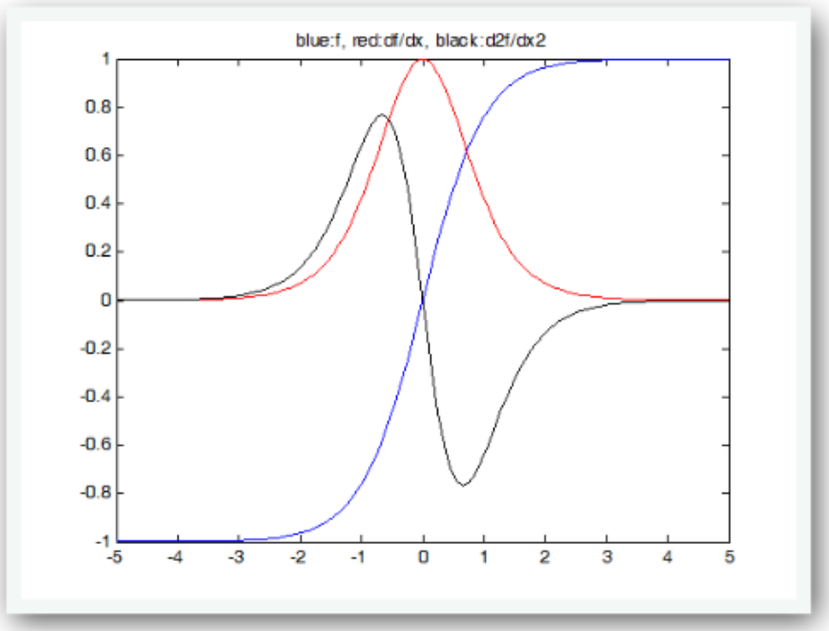
```
K = 2; a = pi/4; da = 10^-2;
x = sym('x');
s{1} = diff(tanh(x));
for i = 2: K
    s{i} = diff(s{i-1});
end
fprintf('tanh(a+da) = %15.14f\n',tanh(a+da));
app =        ;
for k = 1 : K
    f{k} = inline(s{k});
    app = app +                                ;
end
fprintf('app          = %15.14f\n', app);

tanh(a+da) = 0.66145622234088
app        = 0.66145616633167
```

(15 points) Let $f(x) = tanh(x)$. Use diff to create three inline functions.

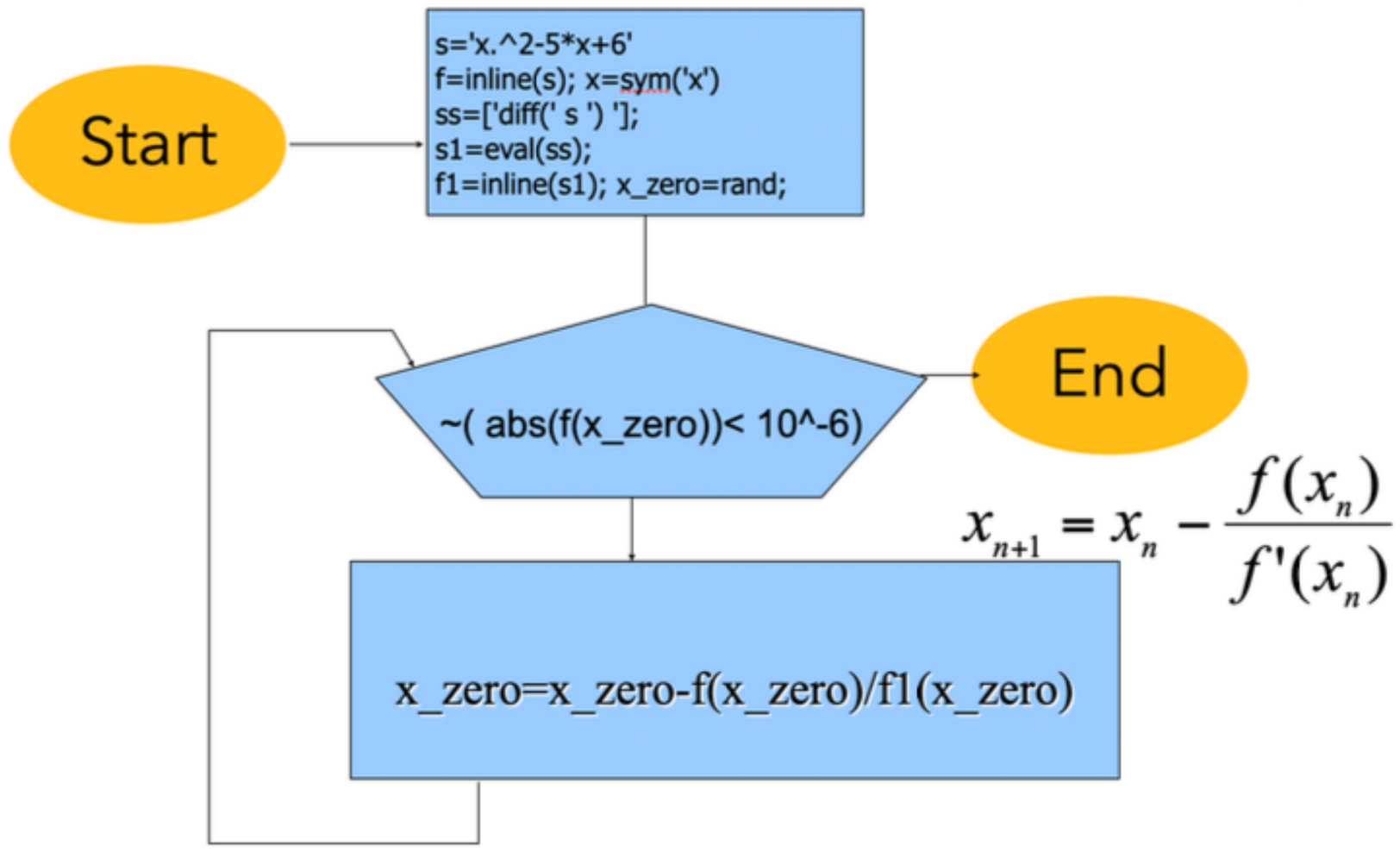
A. (10 points) Create fx, fx1 and fx2, which respectively calculate $f(x)$, $f'(x)$ and $f''(x)$.

B. (5 points) Use these three inline function to plot f, f' and f''.



Newton Method

(10 points) Write a script to implement the following flow chart to find the root.



$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

(15 points) Describe the computational goal of the flow chart in figure 2.

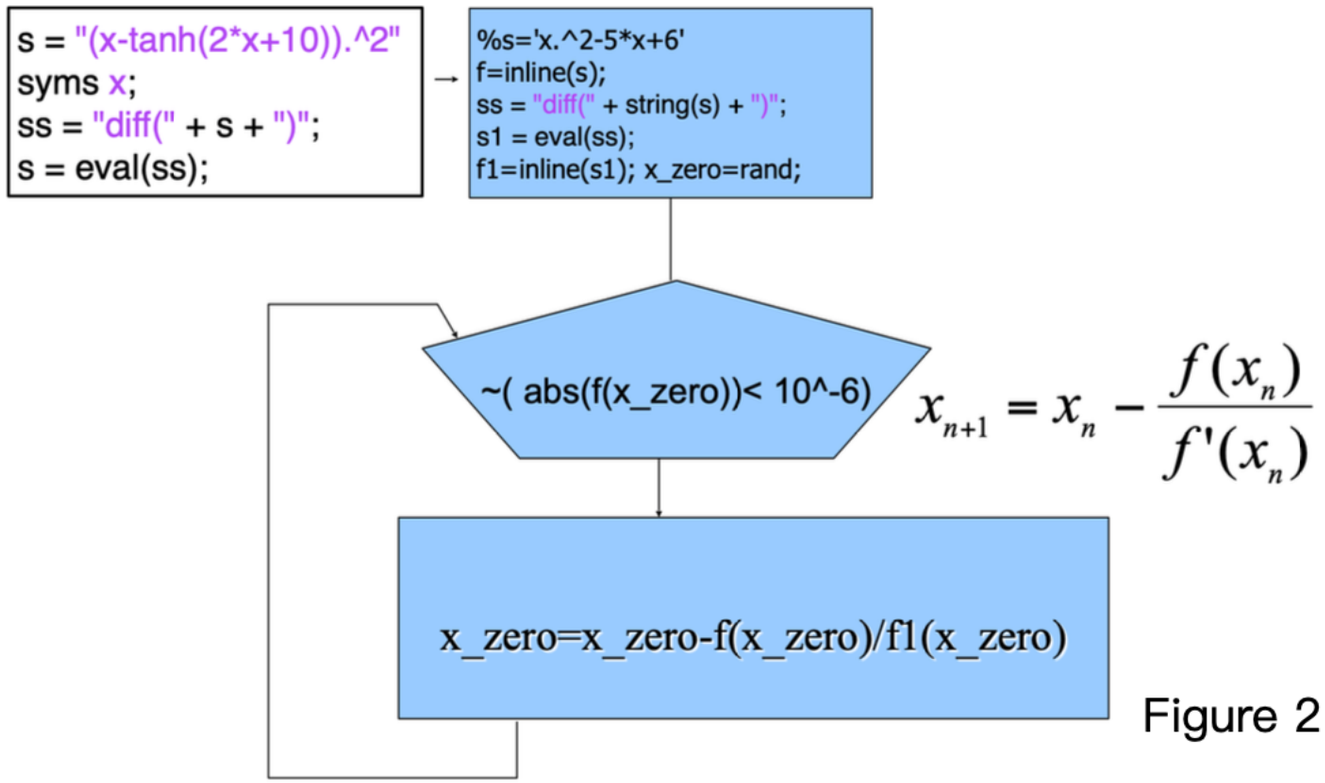
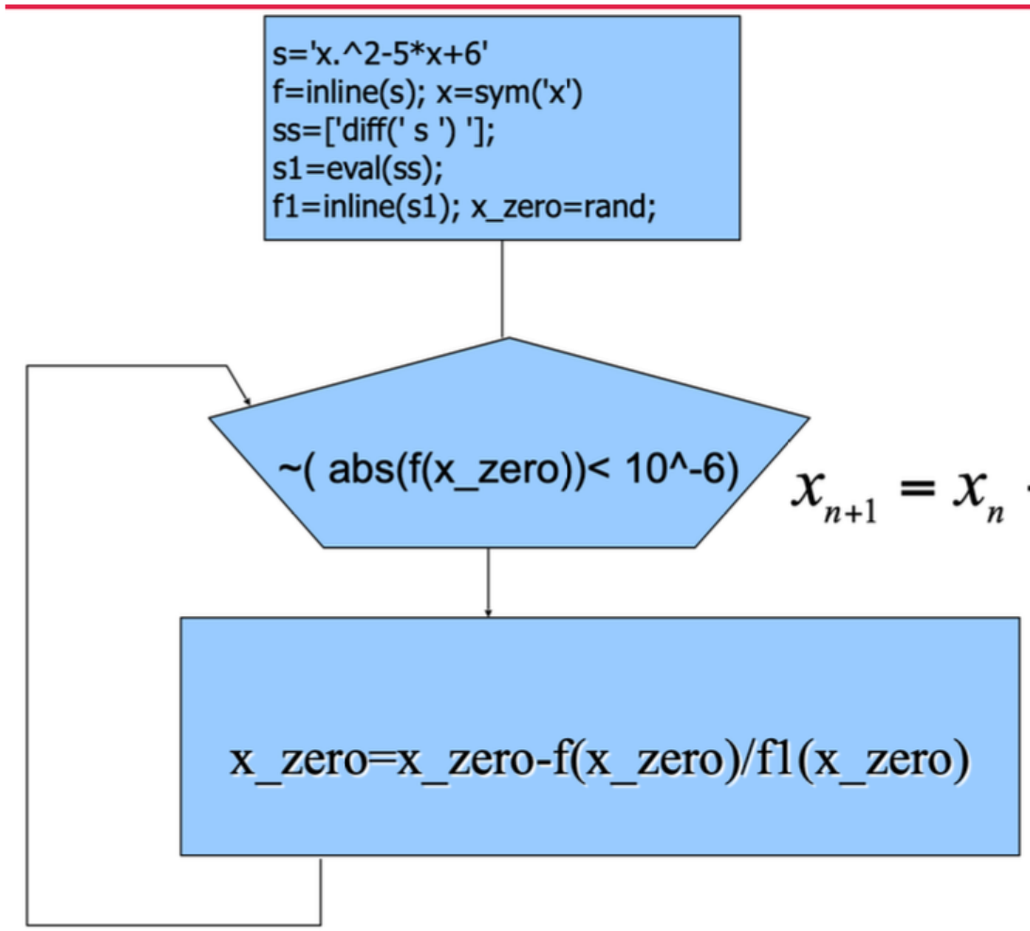


Figure 2

(20 points) Consider the flowchart in figure 1.

A. What is the method realized by the flowchart ?
B. What is the relation between s and x_zero?
C. Explain the halting condition.
D. Derive the updating rule.



$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Figure 1.

Lagrange polynomial

. (10 points) Let L_1, L_2 and L_3 denote three Lagrange polynomial in 2 degree defined by knots 0, 1 and 2.

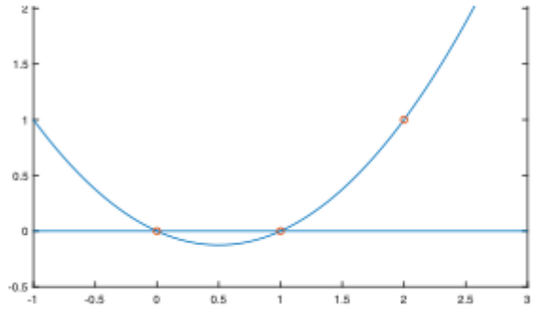
A. (2 points) $L_1(x)$ passes $(0,1)$, $(1,0)$ and (2,0). $L_1(x) = ?$

B. (2 points) $L_2(x)$ passes $(0,0)$, $(1,1)$ and (2,0). $L_2(x) = ?$

C. (2 points) $L_3(x)$ passes $(0,0)$, $(1,0)$ and $(2,1)$. $L_3(x) = ?$

D. (4 points) Complete Matlab codes to plot the following figure.

```
1 -    syms x
2 -    r1 = inline(([redacted])*(x-[redacted])/(2-0)/(2-1));
3 -    z = linspace(-1,3);
4 -    plot(z,[redacted]);hold on
5 -    line([-1 3],[0 0]);
6 -    plot([redacted],[redacted],'o')
```



(10 points) Express three Lagrange polynomials defined by knots 0, 1 and 2.

(10 points) Let g(x) be a polynomial of degree 3. Assume g(–1) = g(0) = g(1) = 0 and g(2) = 1. Please use Matlab function, poly, to find coefficients of g(x).

(10 points) A=reshape(1:16,4,4);a=3;b=1;c=2;

a. A(:,[a b c]) = ?

b. A([a b c],:) = ?

(20 points) The following program finds a circle that passes three random points.

A. (5 points) Write a Matlab

```
function demo_outter_circle()
A = rand(3,2)*4*pi - 2*pi;
x = fsolve(@(x) [redacted], [0;0])
r = norm(x'- A(1,:),2);
hold on
draw_circle(x,r);
plot(A(:,1),A(:,2),'bo');
plot(x(1),x(2),'*');
```

```
function ans = dis(x,A)
    ans(1) = sqrt([redacted])^2 + (x(2)-A(1,2))^2) ;
    ans(1) = ans(1) - sqrt( (x(1)-A(2,1))^2 + ([redacted])^2);
    ans(2) = sqrt((x(1)-A(1,1))^2 + (x(2)-A(1,2))^2) ;
    ans(2) = ans(2) - sqrt([redacted])^2 + (x(2)-A(3,2))^2);
```

function, draw_circle(x,r), to draw a circle that is centered at x with radius r on a plane.

B. (5 points) Complete the Matlab

script.

C. (10 points) Explain why the program is able to find a circle that passes three given points.

(10 points) Write Matlab codes to calculate $57^{\frac{1}{3}}$ by using fsolve.

Find a zero of $f(x) = x^2 - 2x - 3$

```
1    function demo()
2      z = fsolve(                , 0.5)
3
4    function ans = myfun(x)
5      ans =                          ;
```

(20 points) Write equations and codes by filling the following blanks from a to e

a. (4 points) _____

b. (4 points) _____

c. _____

d. _____

e. _____

f. _____

$$\begin{cases} \boxed{\text{a}} \\ x_1 cos(x_2) + x_2 sin(x_1) - 1/2 = 0 \end{cases} (myfun2)$$

$$\begin{cases} 3x_1 - cos(x_2 x_3) - \frac{1}{2} = 0 \\ x_1^2 - 81(x_2 + 0.1)^2 + sin(x_3) = 0 \\ e^{-x_1 x_2} + 20x_3 + \frac{1}{3}(10\pi - 3) = 0 \end{cases} (myfun)$$

$$\begin{cases} \boxed{\text{b}} = 0 \\ x_1^2 - x_2^2 = 0 \end{cases} (myfun1)$$

```
function demo_ex0407()
    x = fsolve(@(x) myfun2(x), rand(2,1))
    myfun2(x)
    x = fsolve(@(x) myfun(x), [redacted c ))
    myfun(x)
    x = fsolve(@(x) myfun1(x), rand(2,1))
    myfun1(x)
function ans [redacted d
    ans(1) = exp(-exp(-(x(1)+x(2)))) - x(2)*(1+x(1)^2);
    ans(2) = x(1)*cos(x(2)) +x(2)*sin(x(1)) - 1/2;
function ans = myfun(x)
    ans(1) = 3*x(1) - cos(x(2)*x(3)) - 1/2;
    ans(2) = x(1)^2 - 81*(x(2)+0.1)^2 + sin(x(3)) + 1.06;
    ans(3) = [redacted e ;
function ans = myfun1(x)
    ans(1) = x(1)^2 + x(2)^2 - 1;
    ans(2) = [redacted f ;
```

(20 points) State how to use the matlab function, fsolve, to solve the following nonlinear system.

$$3x_1 - \cos(x_2 x_3) - \frac{1}{2} = 0$$

$$x_1^2 - 81(x_2 + 0.1)^2 + \sin(x_3) + 1.06 = 0$$

$$e^{-x_1 x_2} + 20x_3 + \frac{1}{3}(10\pi - 3) = 0$$

(10 points) Write Matlab codes to implement the following nonlinear function.

$$\left\{\begin{array}{rcl} f_1(x_1,x_2,x_3) &=& 3x_1 - cos(x_2x_3) - \frac{1}{2} \\ f_2(x_1,x_2,x_3) &=& x_1^2 - 81(x_2+0.1)^2 + sin(x_3) + 1.06 \\ f_3(x_1,x_2,x_3) &=& e^{-x_1x_2} + 20x_3 + \frac{1}{3}(10\pi - 3) \end{array}\right\}$$

(15 points) Write a matlab function to implement the following nonlinear function,

$$\left\{\begin{array}{c} g_1(x_1, x_2) = e^{-e^{-(x_1+x_2)}} - x_2(1+x_1^2) \\ g_2(x_1, x_2) = x_1 cos(x_2) + x_2 sin(x_1) - 1/2 \end{array}\right\}$$

. (5 points) Complete Matlab codes to solve the nonlinear system.

$$\left\{\begin{array}{c} e^{-e^{-(x_1+x_2)}} - x_2(1+x_1^2) = 0 \\ x_1 cos(x_2) + x_2 sin(x_1) - 1/2 = 0 \end{array}\right\}$$

```
function demo_fsolve3()
    x0 = rand(1,2);
    x = fsolve(          );
    x
    y = myfun(x);
    sum(   (y))
end
function y = myfun(x)
    y(1) = exp(-exp(-(x(1)+x(2))))-x(2)*(1+x(1)^2);
    y(2) = x(1)*cos(x(2))+              -1/2;
end
```

Command Window

```
x =

    0.3532    0.6061


ans =

   3.6936e-09
```

Find a zero of $f(x) = x^2 - 2x - 3$

```
1     function demo()
2         z = fsolve(                , 0.5)
3
4     function ans = myfun(x)
5         ans =                          ;
```