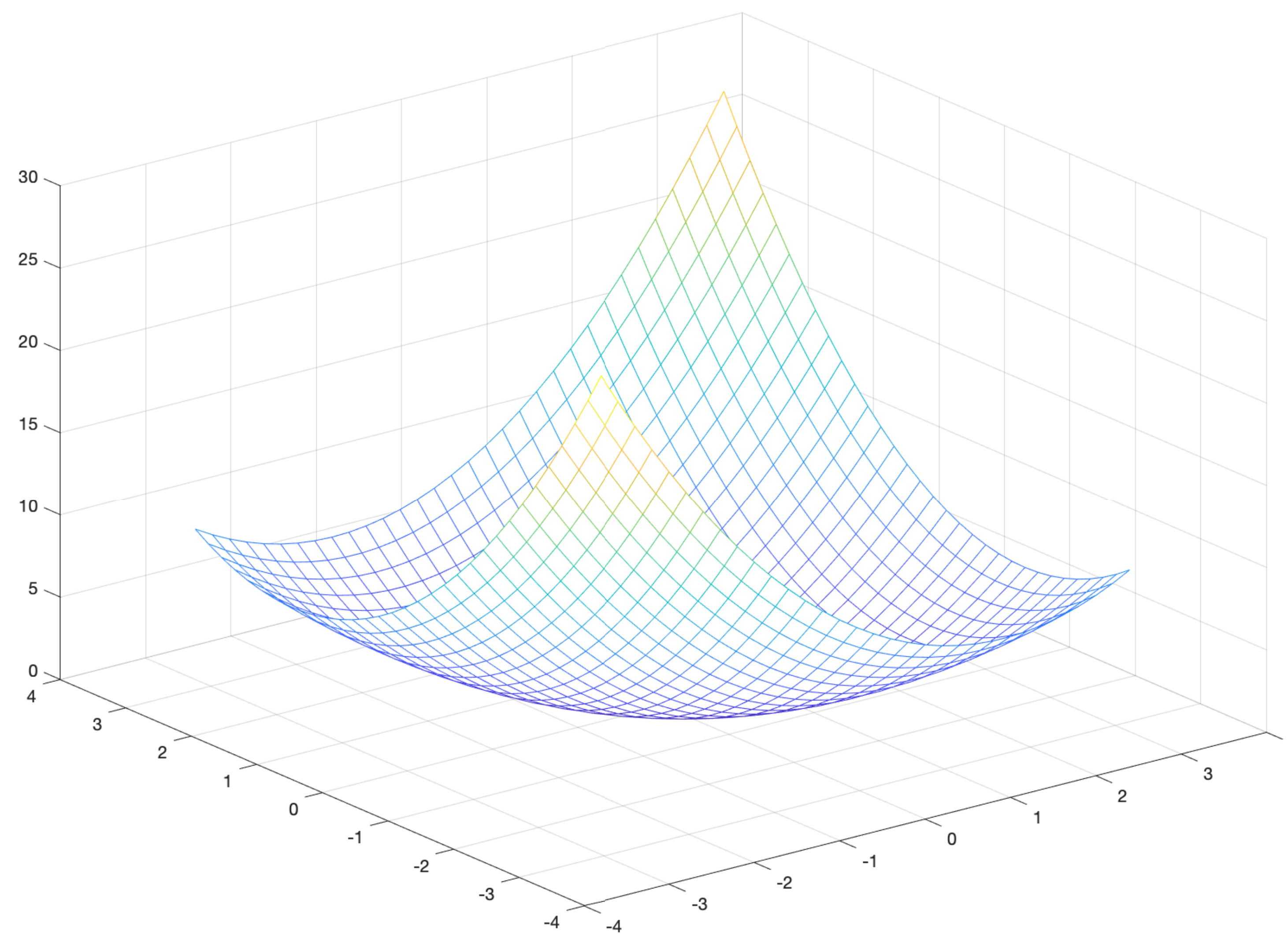
Quadratic surface fitting

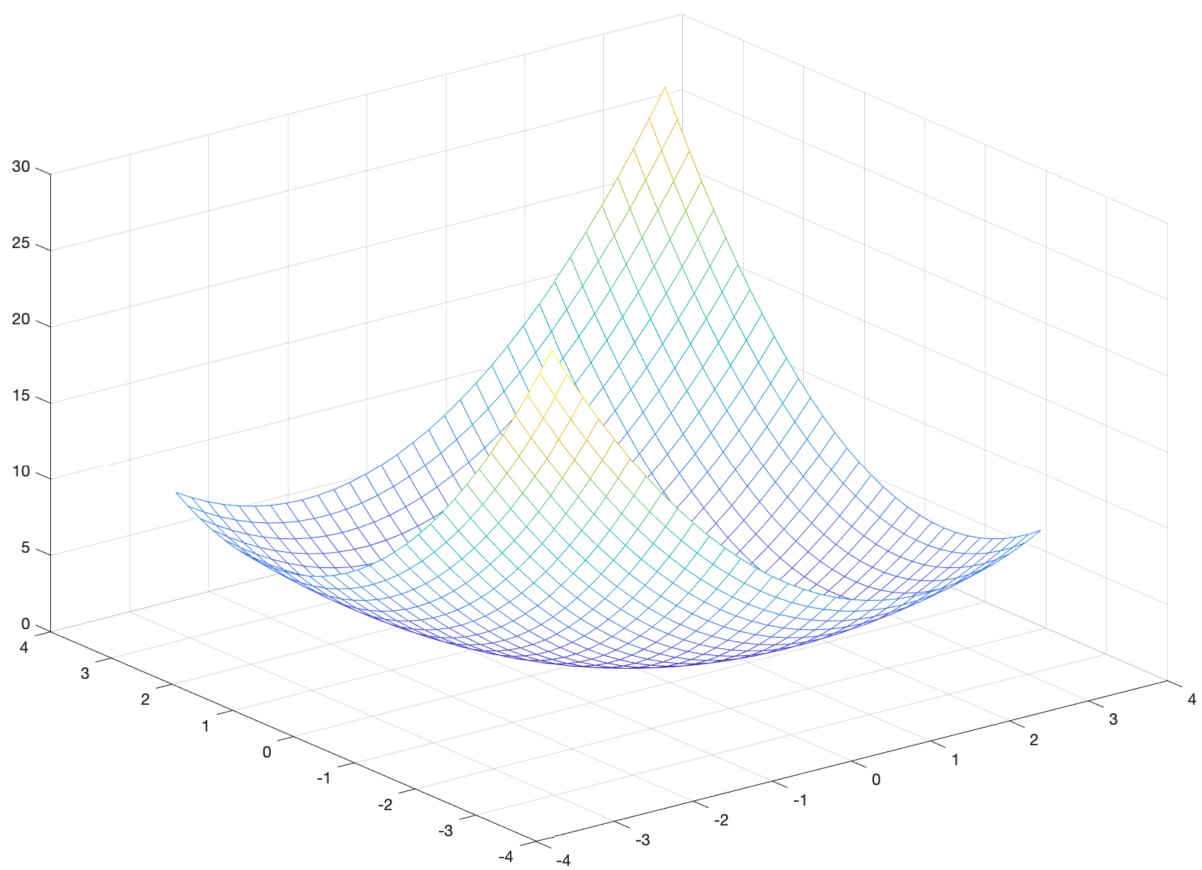
Q surface:

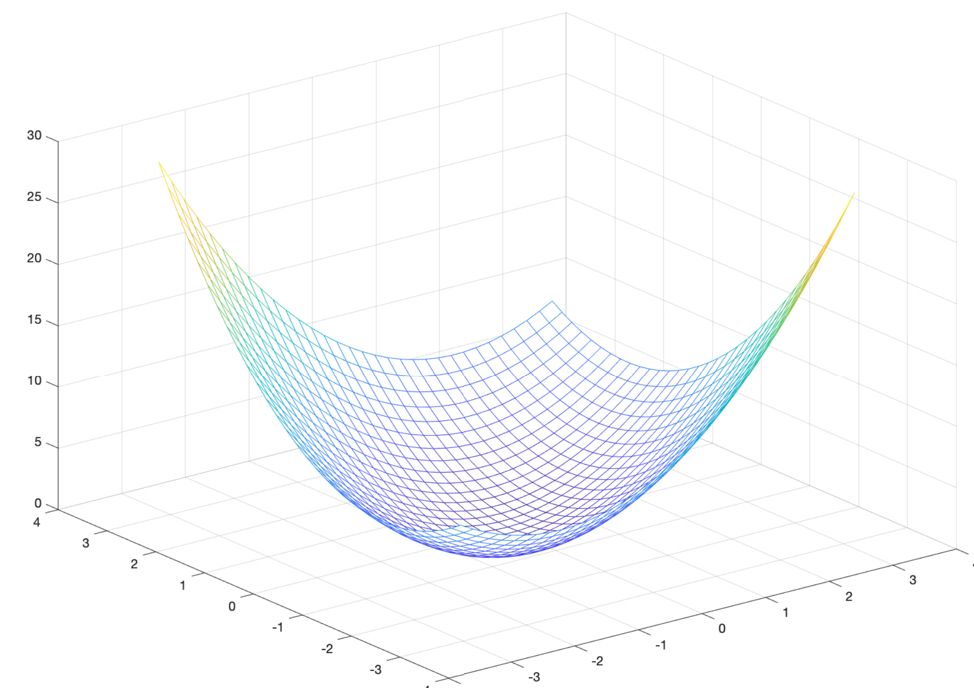
$$x_1^2 + x_1 x_2 + x_2^2$$



Q surface :

$$x_1^2 - x_1 x_2 + x_2^2$$





```
function plot_Q_surface(ss)
% ss = "x1.^2+x1.*x2+x2.^2"
   f = inline(ss);
   x1 = -pi:0.2:pi;
   x2 = x1;
   for i = 1:length(x1)
      y = f(x2,x1(i));
      C(i,:) = y;
   end
   mesh(x1,x2,C);
```

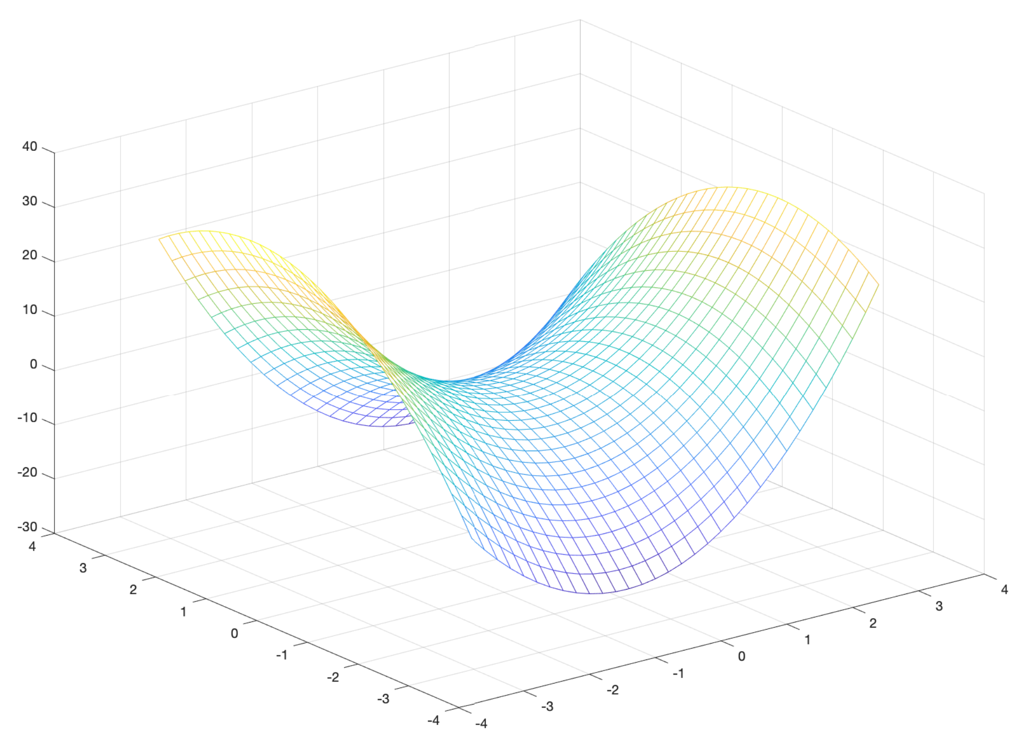
```
>> ss = "x1.^2-x1.*x2+x2.^2";
>> plot_Q_surface(ss)
```

Q-Surface sampling

Q surface :

$$3x_1^2 - 1.5x_1x_2 - 2x_2^2$$

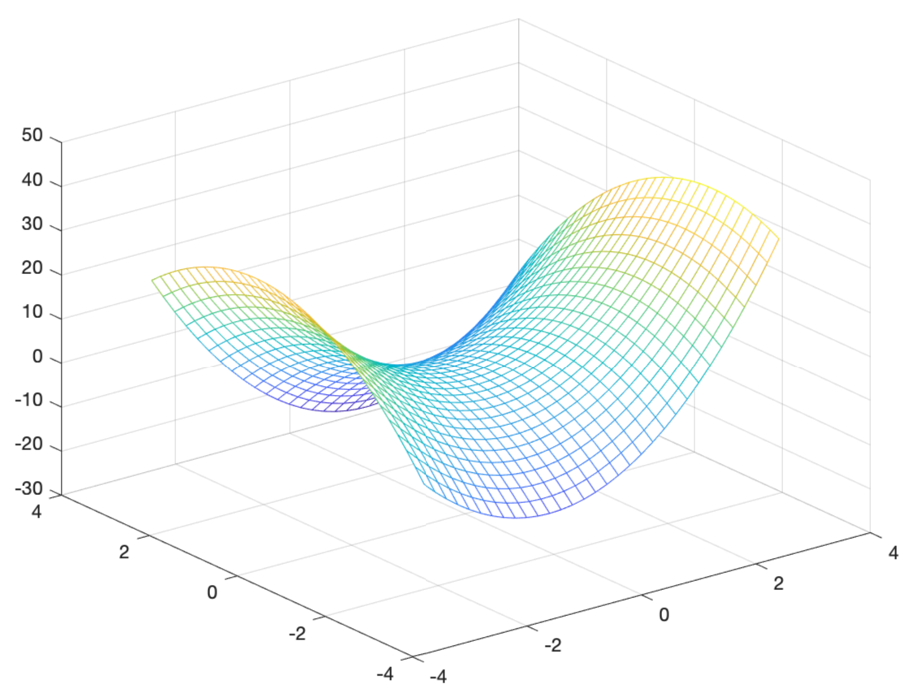
```
>> ss = "3*x1.^2-1.5*x1.*x2-2*x2.^2";
>> plot_Q_surface(ss)
```



Q surface :

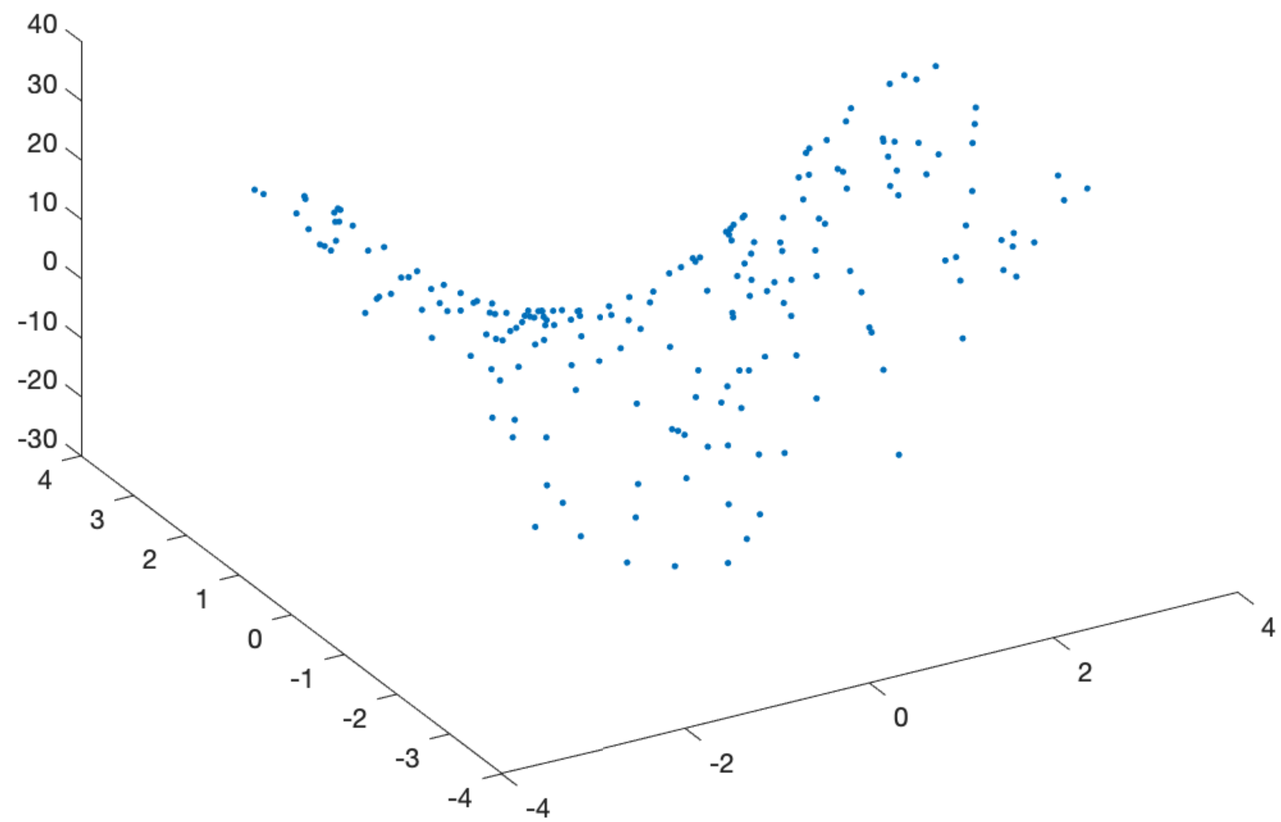
$$3x_1^2 - 1.5x_1x_2 - 2x_2^2 + x_1 - 2x_2 + 4$$

```
>> ss = "3*x1.^2-1.5*x1.*x2-2*x2.^2+x1-2*x2+4";
>> plot_Q_surface(ss)
```



```
function [x y] = sampling_Q_Sur(ss)
% sampling
% ss = "3*x1.^2-1.5*x1.*x2-2*x2.^2+x1-2*x2+4";
    N = 200;
    d = 2;
    x = rand(d,N)*2*pi-pi;
    noise = rand(1,N)*0.2-0.1;
    f = inline(ss);
    y = f(x(1,:),x(2,:));
    y = y + noise;
```

```
>> ss = "3*x1.^2-1.5*x1.*x2-2*x2.^2+x1-2*x2+4";
>> [x,y] = sampling_Q_Sur(ss);
>> plot3(x(1,:),x(2,:),y,'.')
```

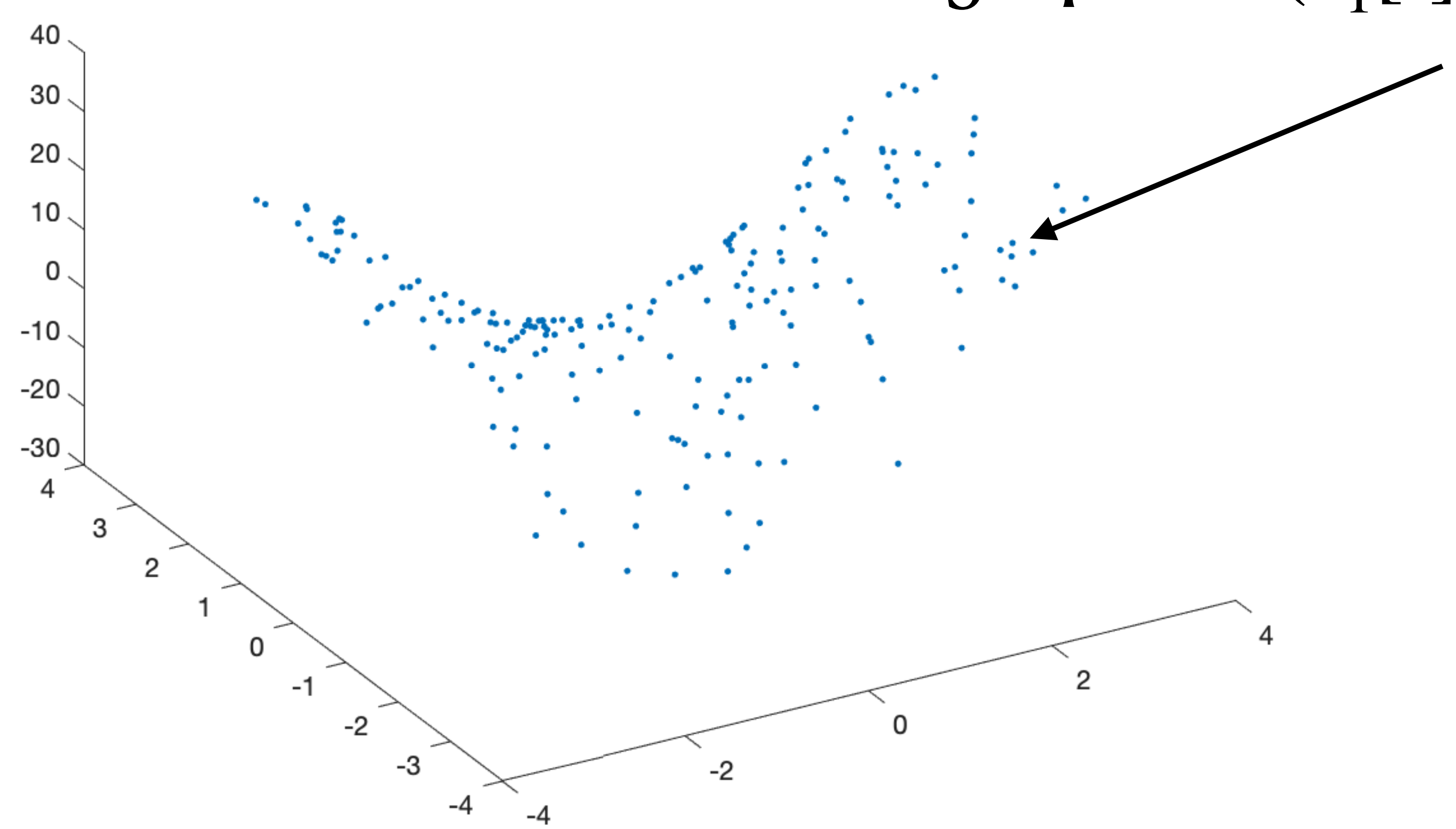


Q surface fitting: Given x and y, find coefficients

Q surface coefficients :

$$c_1 x_1^2 - c_2 x_1 x_2 - c_3 x_2^2 + c_4 x_1 + c_5 x_2 + c_6 = y$$

A 3D point: $(x_1[i], x_2[i], y[i])$



Substitute

$$(x_1[i], x_2[i], y[i])$$

to Q Surface

Linear constraint :

$$c_1 x_1^2[i] - c_2 x_1[i] x_2[i] - c_3 x_2[i]^2 + c_4 x_1[i] + c_5 x_2[i] + c_6 = y[i]$$

All Linear Constraints

$$c_1 x_1^2[1] + c_2 x_1[1] x_2[1] + c_3 x_2[1]^2 + c_4 x_1[1] + c_5 x_2[1] + c_6 = y[1]$$

$$c_1 x_1^2[2] + c_2 x_1[2] x_2[2] + c_3 x_2[2]^2 + c_4 x_1[2] + c_5 x_2[2] + c_6 = y[2]$$

$$\vdots$$

$$c_1 x_1^2[i] + c_2 x_1[i] x_2[i] + c_3 x_2[i]^2 + c_4 x_1[i] + c_5 x_2[i] + c_6 = y[i]$$

$$\vdots$$

$$c_1 x_1^2[N] + c_2 x_1[N] x_2[N] + c_3 x_2[N]^2 + c_4 x_1[N] + c_5 x_2[N] + c_6 = y[N]$$

Find the best

$$c_1, c_2, \ldots, c_6$$

Matrix Form

$$
\begin{bmatrix}
x_1^2[1] & x_1[1]x_2[1] & x_2[1]^2 & x_1[1] & x_2[1] & 1 \\
x_1^2[2] & x_1[2]x_2[2] & x_2[2]^2 & x_1[2] & x_2[2] & 1 \\
 & & & \vdots & & \\
x_1^2[i] & x_1[i]x_2[i] & x_2[i]^2 & x_1[i] & x_2[i] & 1 \\
 & & & \vdots & & \\
x_1^2[N] & x_1[N]x_2[N] & x_2[N]^2 & x_1[N] & x_2[N] & 1
\end{bmatrix}
\begin{bmatrix}
c_1 \\ c_2 \\ \\ c_k \\ \\ c_6
\end{bmatrix}
\begin{matrix}
= y[1] \\ = y[2] \\ \\ = y[i] \\ \\ = y[N]
\end{matrix}
$$

$$Xc = Y$$

$$\begin{bmatrix} x_1^2[1] & x_1[1]x_2[1] & x_2[1]^2 & x_1[1] & x_2[1] & 1 \\ x_1^2[2] & x_1[2]x_2[2] & x_2[2]^2 & x_1[2] & x_2[2] & 1 \\ & & & \vdots & & \\ x_1^2[i] & x_1[i]x_2[i] & x_2[i]^2 & x_1[i] & x_2[i] & 1 \\ & & & \vdots & & \\ x_1^2[N] & x_1[N]x_2[N] & x_2[N]^2 & x_1[N] & x_2[N] & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ \\ c_k \\ \\ c_6 \end{bmatrix} \begin{matrix} = y[1] \\ = y[2] \\ \\ = y[i] \\ \\ = y[N] \end{matrix}$$

$$\mathbf{X} = \begin{bmatrix} x_1^2[1] & x_1[1]x_2[1] & x_2[1]^2 & x_1[1] & x_2[1] & 1 \\ x_1^2[2] & x_1[2]x_2[2] & x_2[2]^2 & x_1[2] & x_2[2] & 1 \\ & & & \vdots & & \\ x_1^2[i] & x_1[i]x_2[i] & x_2[i]^2 & x_1[i] & x_2[i] & 1 \\ & & & \vdots & & \\ x_1^2[N] & x_1[N]x_2[N] & x_2[N]^2 & x_1[N] & x_2[N] & 1 \end{bmatrix}$$

Form matrix X and Y

X is not invertible
Apply pseudo inverse to solve c

X : $N \times 6$

Y: $N \times 1$

c: 6×1

c = pinv(X)*Y

$$Xc = Y$$
$$X^T X c = X^T Y$$
$$c = inv(X^T X) X^T Y$$

Find $\hat{Y}$

$$\hat{Y} = Xc = X(X^TX)^{-1}X^TY$$

Find mean square error

mean(($\hat{Y}$-Y).^2)

Q Surface Fitting

1. Input x and y
2. Form matrix X and Y
3. Find c
4. Find $\hat{Y}$
5. Find mean square error
6. Plot surface